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Research Article

Stability Discussions on some h-type Indexes

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ABSTRACT
The h-index, proposed in 2005, is very popular in the literature and in real-world applications, due to its simplicity and to the importance of measuring scientific impact. Many studies have proposed new indexes with similar characteristics, known as h-type indexes, whose purpose is to overcome certain disadvantages of the original index. This paper studies a few of these indexes that increase the h-index's sensitivity to frequently cited publications, namely the g, A, R and the recently proposed Ψ-index. First, we compare the indexes' responses to increases in the level of academic production, considering a real case study. Moreover, we analyze the indexes' stability, when the citation distribution varies. For a better understanding of the indexes' reactions to such changes, we compare them to the p and z indexes.

Keywords: Decision processes, h-index, Bibliometric indexes, Comparisons between indexes, h-type indexes, Index Stability.

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INTRODUCTION

Hirsch's[1] bibliometric index, called the *h*-index, is one of the most popular indicators for measuring the impact and relevance of scientific production, probably due to its simplicity and easy understanding.[2] Not only has the *h-index* been much studied, but also widely used, in several areas associated with scientific research.[3]

Among hundreds of studies related to the *h*-index, many present criticisms, e.g. Bornmann and Daniel,[4] Braun *et al*.[5] Burrell,[6] Harzing and Van der Wal,[7] Norris and Oppenheim,[8] Panaretos and Malesios,[9] and propose alternative indices to overcome disadvantages, e.g.., Alonso *et al*.[10] Anderson *et al*.[11] Batista *et al*.[12] Cabrerizo *et al*.[13] Egghe and Rousseau,[14] Schreiber,[15] Tol,[16] Vinkler,[17,18] Zhang,[19] and even Hirsch[20,21] himself, among many other studies.

Despite the fact that there is over one hundred indicators, most organizations still evaluate researchers by their *h*-index alone.[22] This indicator ignores exceptional papers, even though it is precisely with these paper that researchers win prizes.[16] From this perspective, certain educational institutions and funding agencies may prefer to evaluate their candidates based on different bibliometric indexes. However, to decide upon the most suitable indicator for their purposes, these organizations should fully understand how these indicators respond to different types of researcher profiles.

Therefore, this paper studies and compares some h-type indexes that increase the *h*-index's sensitivity to frequently cited publications, namely the g-index,[23] the A-index,[24] the R-index[25] and the recently proposed Ψ-index (Lathabai, 2020).[26] First, we study the effects of citation increases, based on a real case study of university professors, considering two distinct periods. Then, we examine the indexes' stability to variations in the citation distribution, based on a sensitivity analysis. We find that there are still some discussions on how indicators, particularly the g-index, respond to citation redistributions and we intend to help clarify this with our study.

For a better understanding of the indexes' reactions to such changes, we compare them to the p-index[27,28] and the z-index.[29] The first balances a researcher's activity and excellence, whereas the latter combines quantity, quality and consistency of research performance. Thus, we find that these indicators help us interpret results.

REVIEW OF SOME BIBLIOMETRIC INDEXES

H-Index

Hirsch[1] proposed the *h*-index to evaluate the impact and relevance of scientific production, in a simple and intuitive way. By definition, a researcher presents an index of h, if he/she authors h publications with at least h citations each, while the other publications present no more than h citations. The h publications with at least h citations each form the so-called h-core.

Many studies pointed out disadvantages of this index, e.g. the fact that it does not consider the age of publications, which benefits experienced researchers,[30] and that it is insensitive to widely cited publications.[3,23,31]

Some criticisms led to new bibliometric indexes that consider other variables, besides the number of publications and citations, such as the age of publication. The indices that only consider the variables originally proposed by Hirsch,[1] are called h-type indexes.[3] In the present study, we analyze certain h-type indexes, which overcome the *h*-index's insensitivity to frequently cited publications.

G-INDEX

One of the most popular variants of the *h*-index,[2] the g-index is defined as the largest natural number of publications that together received at least g^2 citations,[23] which form the so-called g-core. In other words, the g-index is equivalent to the number of publications cited, on average, at least g times.[31] Thus, this index values publications that are widely cited and is always greater or equal to the *h*-index.[3] Brandão and Soares de Mello[32] show in which situations such indices are equivalent.

Among the g-index's criticisms, the data accuracy issue stands out,[25] because its core is usually larger than that of the *h*-index, thus depending on a larger set of accurate information. Thus, the A-index was proposed to deal with the aforementioned insensitivity of the *h*-index, without adding accuracy problems.

A-INDEX

The A-index is the h-core's average number of citations,[24] as shown in (1), where h is the number of publications in the h-core and c_j is the number of citations for each publication j=1,...,h, where A ≥ h.[25]

$$A = \frac{\Sigma_{j=1}^{h} c_j}{h} \tag{1}$$

This index has a certain similarity with the g-index, as both are the average number of citations among a set of publications, although the g-index is a natural number and the A-index is a real number. In addition, the sets of publications are different: the A-index considers the h-core, whereas the g-index has its own core. Jin *et al.*[25] shows that A ≥ g.

However, the A-index does not increase monotonically with citation increases. In other words, it is possible that new citations reduce a researcher's A-index. This may occur when new citations increase the researcher's *h*-index and the new core has a lower citation average. This undesirable characteristic is consequence of the division by h.[25] To deal with this issue, Jin *et al.*[25] Proposed the R-index.

R-INDEX

The R-index is the root of the total number of citations in the h-core,[25] as shown in (2), where the variables are the same as in (1).

$$R = \sqrt{\Sigma_{j=1}^{h} c_j} \tag{2}$$

As (2) is not a division by h, the R-index increases monotonically with the number of citations. Moreover, the R-index is sensitive to widely cited publications. According to Jin *et al.*[25] we have R ≥ h and A ≥ R. However, it is not possible to define a relationship between g and R, as explained in Brandão and Soares de Mello.[32]

Ψ-INDEX

On the other hand, Lathabai[26] understands that top papers with massive citations should have the power to *offset* relatively less performing papers, which the author called offset-ability. The g-index reflects such offset-ability, though not to the greatest possible extent. This is why Lathabai[26] proposed the ψ-index, defined as the highest position such that citations earned by top ψ papers average at least to $\frac{(\psi+1)}{2}$. To elucidate this definition, we should add that this index is the solution to (3) and (4), where the variables are the same as in previous equations.

$$\max_{\Psi} \Sigma_{j=1}^{\Psi} j \leq \Sigma_{j=1}^{\Psi} c_j \tag{3}$$

$$\max_{\Psi} \frac{\Psi(\Psi+1)}{2} \leq \Sigma_{j=1}^{\Psi} c_j \tag{4}$$

The author shows that $\Psi \geq g$ and that the average citations in the -core, a_ψ, compared to the average citations in the g core, a_g, is $a_\psi \leq a_g \cong g$.

P-INDEX

With different purposes, the p-index introduced by Prathap[27] and renamed by Prathap,[28] balances the total number citations (activity) and the number of citations per publication (excellence). Its value is computed as (5), where $C = \Sigma_{j=1}^{P} c_j$ is the total number of citations and *P* is the total number of publications:

$$p = \left(\frac{C^2}{P}\right)^{\frac{1}{3}} \tag{5}$$

Prathap[33,34] called $X = \frac{C^2}{P}$ a robust second-order indicator, representing quality (velocity) and quantity (volume). This index does not increase monotonically with the number of publications, because it is divided by *P*.

Z-Index

Based on a similar perspective, Prathap[29] proposed a three dimensional evaluation combining the attributes of quality, quantity and consistency into a single index, where $\eta = \frac{X}{\sum c_j^2}$ represents consistency:

$$z = (\eta . X)^{\frac{1}{3}} \quad (6)$$

We may also observe that this index does not increase monotonically either.

INCREASE IN ACADEMIC PRODUCTION – CASE STUDY

In this section, we analyze how the aforementioned indexes behave when there is an increase in the researchers' production level, considering the academic output of professors in the Production Engineering postgraduate course at UFF.

MATERIAL AND METHODS

Herein, we use the study conducted in De Castro Reis *et al.*[35] as the initial basis. These authors identified professors who worked in the aforementioned postgraduate course, from October 19 to October 31, in 2016, by consulting the department's website. Then, they obtained the professors' *h*-index, from the Scopus database. In the present case study, we performed the same procedure, on May 10, 2019, i.e., approximately two and a half years later.

We consider that professors who are not listed in the Scopus database did not publish any study in a relevant vehicle for the scientific community, or that their work did not receive citations. In either case, such professors were disregarded from the present study. Tables 1-3 present the number of publications and citations for each professor in our case study. It should be noted that, to avoid possible discomforts, we do not to disclose their names.

RESULTS AND DISCUSSION

Table 4 presents the values for the indexes studied herein, based on the information from Tables 1 to 3.

Only 2 of 17 professors maintained the same *h*-index over this period (PRO12 and PRO17). PRO17 did not publish new cited articles and received new citations for a single publication. PRO12 published a new cited article and received a new citation. In both cases, the h and g indexes did not vary, but the A, R and p indexes did. In fact, the R and the p index increased for all professors. Perhaps, the fact that these are real numbers and not natural ones, such as h and g, explains this susceptibility to minor variations.

Even though the ψ-index is a natural number, it managed to capture PRO17's improvement. This illustrates how the ψ-index actually enhances g's power to reflect the offset-ability. As the ψ index did not increase for PRO12, we could interpret that his/her improvement was not sufficient. In fact, the z index for this professor actually decreased slightly, because of his/her loss in consistency.

Table 1: Number of publications and citations for the first six professors.

Publication Order	PRO01		PRO02		PRO03		PRO04		PRO05		PRO06	
	2016	2019	2016	2019	2016	2019	2016	2019	2016	2019	2016	2019
1	103	144	147	212	24	42	10	42	104	144	16	43
2	33	40	75	95	21	37	8	27	38	47	8	22
3	27	34	51	90	20	36	8	25	33	40	8	17
4	25	30	44	86	18	29	7	17	27	34	5	14
5	21	27	29	53	11	26	4	15	21	25	4	12
6	17	25	24	42	10	26	3	12	17	24	3	11
7	17	24	21	37	9	23	2	11	17	21	2	10
8	16	21	20	36	9	22	2	10	12	17	2	10
9	12	17	18	31	9	21	2	8	11	17	2	8
10	11	17	17	29	7	21	2	8	10	16	2	8
11	10	16	15	26	6	20	1	7	9	14	2	7
12	9	14	14	26	6	19	1	7	9	14	1	6
13	9	14	14	24	5	14	1	6	9	14	1	6
14	9	14	13	23	4	13	-	6	8	12	1	6
15	8	14	13	23	4	13	-	6	8	12	1	5
16	8	12	12	22	4	12	-	6	7	11	1	4
17	7	11	12	21	3	12	-	5	7	11	1	4
18	7	11	11	21	3	11	-	5	6	11	1	4

continued...

Table 1: Cont'd.

Publication Order	PRO01		PRO02		PRO03		PRO04		PRO05		PRO06	
	2016	2019	2016	2019	2016	2019	2016	2019	2016	2019	2016	2019
19	7	11	11	19	3	11	-	4	6	11	1	4
20	7	11	11	19	2	10	-	4	5	10	1	4
21	6	10	10	18	2	10	-	4	5	8	1	3
22	6	9	10	16	1	10	-	4	4	8	-	3
23	6	9	10	16	1	8	-	4	4	8	-	3
24	6	8	9	16	1	8	-	4	4	8	-	3
25	6	8	9	15	-	7	-	3	4	7	-	2
26	5	8	7	14	-	7	-	3	3	6	-	2
27	5	8	7	14	-	6	-	3	3	6	-	2
28	5	8	6	14	-	6	-	3	2	6	-	2
29	4	7	6	13	-	5	-	3	2	5	-	2
30	4	7	4	12	-	5	-	3	1	5	-	2
31	4	7	4	12	-	5	-	3	1	5	-	1
32	4	7	4	12	-	5	-	2	1	5	-	1
33	4	7	4	11	-	5	-	2	-	4	-	1
34	4	6	4	11	-	4	-	2	-	4	-	1
35	3	6	4	11	-	4	-	2	-	4	-	1
36	3	6	4	10	-	4	-	2	-	3	-	1
37	3	6	3	9	-	4	-	2	-	3	-	1
38	2	5	2	8	-	4	-	2	-	2	-	1
39	2	5	2	8	-	3	-	1	-	2	-	1
40	2	5	2	8	-	3	-	1	-	2	-	1
41	2	5	2	6	-	2	-	1	-	2	-	-
42	2	5	2	5	-	2	-	1	-	2	-	-
43	2	5	1	5	-	2	-	1	-	2	-	-
44	2	4	1	5	-	2	-	1	-	2	-	-
45	2	4	1	5	-	1	-	1	-	2	-	-
46	2	4	1	4	-	1	-	1	-	1	-	-
47	1	4	1	4	-	1	-	1	-	-	-	-
48	1	4	-	4	-	1	-	1	-	-	-	-
49	1	4	-	3	-	1	-	1	-	-	-	-
50	1	3	-	2	-	1	-	1	-	-	-	-
51	1	3	-	2	-	1	-	-	-	-	-	-
52	1	3	-	2	-	1	-	-	-	-	-	-
53	1	3	-	2	-	-	-	-	-	-	-	-
54	1	3	-	1	-	-	-	-	-	-	-	-
55	-	3	-	1	-	-	-	-	-	-	-	-
56	-	3	-	1	-	-	-	-	-	-	-	-
57 and 58	-	2	-	1	-	-	-	-	-	-	-	-
59 to 67	-	2	-	-	-	-	-	-	-	-	-	-
68 to 77	-	1	-	-	-	-	-	-	-	-	-	-
TOTAL publications	54	77	47	58	24	52	13	50	32	46	21	40
TOTAL citations	467	716	692	1237	183	547	51	294	398	617	64	239

Table 2: Number of publications and citations for six other professors.

Publication Order	PRO07		PRO08		PRO09		PRO10		PRO11		PRO12	
	2016	2019	2016	2019	2016	2019	2016	2019	2016	2019	2016	2019
1	37	53	21	27	14	28	75	99	5	14	6	7
2	5	13	12	12	4	6	8	16	1	8	2	2
3	4	10	10	11	2	4	6	9	1	4	2	2
4	4	7	9	11	1	2	4	7	1	4	1	1
5	3	6	2	5	-	2	3	5	1	4	-	1
6	2	6	2	5	-	2	3	5	-	3	-	-
7	1	6	2	5	-	2	2	4	-	2	-	-
8	1	6	2	4	-	1	1	4	-	2	-	-
9	1	4	1	3	-	1	1	3	-	2	-	-
10	1	3	1	2	-	1	1	2	-	1	-	-
11	1	3	-	2	-	1	1	1	-	1	-	-
12	-	2	-	2	-	1	1	1	-	-	-	-
13	-	2	-	2	-	1	1	1	-	-	-	-
14	-	1	-	1	-	1	-	1	-	-	-	-
15	-	1	-	1	-	-	-	-	-	-	-	-
16 to 18	-	1	-	-	-	-	-	-	-	-	-	-
TOTAL Publications	11	18	10	15	4	14	13	14	5	11	4	5
TOTAL Citations	60	126	62	93	21	53	107	158	9	45	11	13

Table 3: Number of publications and citations for the last five professors.

Publication Order	PRO13		PRO14		PRO15		PRO16		PRO17	
	2016	2019	2016	2019	2016	2019	2016	2019	2016	2019
1	10	20	24	31	1	3	6	9	5	8
2	-	7	1	12	1	2	1	3	-	-
3	-	7	-	4	1	1	-	2	-	-
4	-	2	-	3	0	1	-	1	-	-
5	-	1	-	-	-	-	-	-	-	-
TOTAL Publications	1	5	2	4	4	4	2	4	1	1
TOTAL Citations	10	37	25	50	3	7	7	15	5	8

The z index for PRO09 also had a small reduction, even though all the other indexes increased. Almost half of all 32 citations that this professor received from 2016 to 2019 were related to his/her single top publication. This indicates that even though the other bibliometric indexes are comparatively high, this professor lost consistency.

To allow further analysis, in Table 5, we present the indexes' relative variations, $V_i = \frac{(i_{2019} - i_{2016})}{i_{2016}}$, for $i = h, g, A, R, \psi, p, z$.

As we previously mentioned, the A-index may present a significant reduction, even when a researcher receives more citations, particularly when his/her *h*-index increases. This happened with PRO14, who published new articles and received new citations over the time period. His/her *h*-index tripled, whereas his/her A-index reduced 35%. This example illustrates that, in fact, A does not increase monotonically with publications and citations. Besides, the h, g and R indexes for PRO16 increased from 2016 to 2019, but the A index did not. In 2016, this professor had 1 publication in the h-core, with 6 citations; and in 2019, he/she had 2 publications, with 12 citations in total. Thus, the aforementioned professor has A = 6 in both cases, even though in 2019 he/she had more publications and citations than in 2016.

Even when A does increase, it is usually at a completely different rate, compared to other indexes. For instance, PRO15 had the greatest A-index increase and only moderate increases with regard to the other indexes. On the other hand, PRO11

Table 4: Values of the h, g, A, R, ψ, p and z indexes for each professor in both years.

Professors	2016							2019						
	h	g	A	R	ψ	p	z	h	g	A	R	ψ	p	z
PRO01	10	18	28,2	16,8	28	15,93	10,13	14	23	31,2	20,9	34	19,25	11,88
PRO02	13	25	37,6	22,1	36	21,68	14,10	19	33	48,2	30,3	48	29,93	20,21
PRO03	9	12	14,6	11,4	18	11,17	9,28	13	20	25,8	18,3	30	17,92	14,36
PRO04	4	6	8,3	5,7	9	5,85	5,00	8	14	19,9	12,6	21	12,00	8,65
PRO05	10	18	29,0	17,0	27	17,04	11,49	13	23	32,8	20,7	33	20,23	13,13
PRO06	4	4	9,3	6,1	9	5,80	4,34	8	13	17,4	11,8	19	11,26	8,17
PRO07	4	7	12,5	7,1	10	6,89	4,20	6	10	15,8	9,7	15	9,59	6,17
PRO08	4	7	13,0	7,2	10	7,27	5,73	5	9	13,2	8,1	13	8,32	6,46
PRO09	2	4	9,0	4,2	6	4,80	3,83	3	6	12,7	6,2	9	5,85	3,61
PRO10	4	10	23,3	9,6	14	9,59	5,12	5	12	27,2	11,7	17	12,13	6,76
PRO11	1	2	5,0	2,2	3	2,53	2,08	4	6	7,5	5,5	8	5,69	4,68
PRO12	2	3	4,0	2,8	4	3,12	2,73	2	3	4,5	3,0	4	3,23	2,69
PRO13	1	3	10,0	3,2	4	4,64	4,64	3	6	11,3	5,8	8	6,49	5,30
PRO14	1	5	24,0	4,9	6	6,79	5,53	3	7	15,7	6,9	9	8,55	7,02
PRO15	1	1	1,0	1,0	2	1,31	1,19	2	2	2,5	2,2	3	2,31	2,15
PRO16	1	2	6,0	2,4	3	2,90	2,53	2	3	6,0	3,5	5	3,83	3,22
PRO17	1	2	5,0	2,2	2	2,92	2,92	1	2	8,0	2,8	3	4,00	4,00

Table 5: Relative variations of the h, g, A, R, ψ, p and z indexes.

Professors	Relative Variations						
	h	g	A	R	ψ	p	z
PRO01	0,40	0,28	0,11	0,24	0,21	0,21	0,17
PRO02	0,46	0,32	0,28	0,37	0,33	0,38	0,43
PRO03	0,44	0,67	0,78	0,60	0,67	0,60	0,55
PRO04	1,00	1,33	1,41	1,20	1,33	1,05	0,73
PRO05	0,30	0,28	0,13	0,21	0,22	0,19	0,14
PRO06	1,00	2,25	0,88	0,94	1,11	0,94	0,88
PRO07	0,50	0,43	0,27	0,38	0,50	0,39	0,47
PRO08	0,25	0,29	0,02	0,13	0,30	0,14	0,13
PRO09	0,50	0,50	0,41	0,45	0,50	0,22	-0,06
PRO10	0,25	0,20	0,17	0,21	0,21	0,27	0,32
PRO11	3,00	2,00	0,50	1,45	1,67	1,25	1,24
PRO12	0,00	0,00	0,13	0,06	0,00	0,04	-0,02
PRO13	2,00	1,00	0,13	0,84	1,00	0,40	0,14
PRO14	2,00	0,40	-0,35	0,40	0,50	0,26	0,27
PRO15	1,00	1,00	1,50	1,24	0,50	0,76	0,81
PRO16	1,00	0,50	0,00	0,41	0,67	0,32	0,27
PRO17	0,00	0,00	0,60	0,26	0,50	0,37	0,37
Average	0,83	0,67	0,41	0,55	0,60	0,46	0,40

had large increases for the other indexes and a very small increase for A. This is why this index should not stand alone and for this reason, we will not study this indicator henceforward.

From Table 5, we may observe that the *h*-index presents the largest variations. PRO11 managed to quadruplicate his/her *h*-index, whereas PRO13 and PRO14 managed to triplicate their indexes. These professors had an *h*-index of 1 in 2016. PRO11 also presented the greatest increase of the R, ψ, z and p indexes, though their variations were roughly around 1,5. Naturally, these indexes were higher than 1 in 2016.

The g-index for this professor triplicated, but this wasn't the largest variation for this index. PRO06 had an increase of 2,25 times his/her g-index. Although the other indexes also increased significantly for this professor, they were only multiplied by 1,0, approximately. From Table 4, we may observe that this professor had g = h = 4, whereas the other indexes were well above this value. This initial state probably explains why the g index increased as much as it did.

On the other hand, those who presented high *h*-index's in 2016, particularly PRO01 (h = 10), PRO02 (h = 13), PRO03 (h = 9) and PRO05 (h=10), as shown in Table 4, had a small index increase, i.e., less than 50%. These professors also presented the highest values for g, R, ψ, z and p, in 2016. For PRO03, these indexes increased around 60%, though for PRO01, PRO02 and PRO05, the variations for these indexes were smaller than for the *h*-index.

On average, the *h-index* presented the highest variation, 83%, followed by the g-index, with 67%, then by the ψ-index, with 60% and finally by the R-index, with 55%. The other indexes varied less than 50%, on average. However, these values are influenced by extreme cases. When we remove those that we mentioned previously, the average increase for PRO04,

PRO06 to PRO10, PRO12 and PRO15 to PRO17, is approximately 50% for h, g, R and ψ and around 40% for p and z.

In short, we find that in the beginning of a researcher's career, the *h*-index tends to increase rapidly, but slows down towards the end. On the other hand, the R, ψ, z and p indexes present more constant increases. The g index stands between both, varying less than the *h*-index, though more than the others, in the beginning.

Consequently, these indexes produce different rankings, which means that organizations would select different researchers, depending on the indicator they use. Thus, it is very important for such organizations to fully understand the indices' characteristics, which includes their stability.

INDEX STABILITY

The indices' stability corresponds to their behavior when facing changes in the citation distribution. Such changes are not possible in practice, for a given researcher. However, the citation distribution is an essential aspect of bibliometric indexes. In fact, Brandão and Soares de Mello (2019)[36] showed that the citation distribution is formally considered a criterion for the *h*-index.

Particularly, the h and g indexes are maximized by a squared distribution, as well as R and z, according to Lando and Bertolli-Barsotti (2014).[37] On the other hand, Wildgaard *et al.* (2014)[22] affirm that g ignores the citation distribution, as it is based on the arithmetic average. To overcome precisely this alleged disadvantage, though without the *h*-index's drawback of ignoring exceptional papers, Tol (2009)[16] proposed intermediary indexes, based on the geometric average number of citations (t) and on the harmonic average (f), where $h \leq f \leq t \leq g$.

Diversely, Lathabai (2020)[26] considers that the possibility of increasing the g index with one or few "big hit" papers is, as a matter of fact, an advantage. Based on this perspective, the author proposed the ψ-index, that enhances this effect. Vinkler[17] also acknowledges the importance of high impact papers, which is why his π-index focuses precisely on these publications.

Therefore, there are multiple discussions on how bibliometric indicators should ideally react to different types of citation distribution. Consequently, the indexes proposed in the literature present different stability features. Besides, there are also discussions on how the indices react to citation redistributions, particularly with regard to the g-index. Thus, in this section, we analyze the indexes studied herein, with regard to this characteristic.

METHOD

To analyze the indexes' stability, the citations of four professors, specifically PRO5, PRO09, PRO10 and PRO11, from Tables 1-3, were first dissipated, i.e., distributed among publications in a more uniform manner, maintaining the same number of total citations and cited publications, so that the *h*-index is maximized. As there may be certain citations whose allocation is indifferent to the researcher's *h-index*, we redistribute citations so that the other indexes are also maximized, as a second objective.

Subsequently, their citations were concentrated, i.e., redistributed so that fewer publications presented more citations, though maintaining the total number of citations and publications. For that, we defined five different scenarios, with the following concentration patterns for the first five publications, in terms of percentage of total citations: 80-5-5-5-5, 20-20-20-20-20, 30-30-30-5-5, 50-30-10-5-5 and 60-20-10-5-5. In addition, we attributed 1 citation to the sixth publication onward.

RESULTS AND DISCUSSION

In Table 6, we present the aforementioned citation dissipation for professors PRO05, PRO09, PRO10 and PRO11.

We may observe that, in Table 6, $h = g \cong R$ for all researchers. In fact, these indices are not exactly the same only because of a few citations that do not influence the *h*-index, but affect $R \in \mathbb{R}$. The ψ-index is significantly greater than these indexes for all cases, whereas p and z are slightly smaller, in general. The p-index remained the same as for the original data because it only depends on the total number of citations and publications, thus, it does not vary with citation concentration or dissipation. On the other hand, the z-index presented a significant increase, indicating that it values dissipated citation distributions.

Moreover, the g and ψ-index for the original and modified data is the same for three out of four professors (PRO09, PRO10 and PRO11) and very close for PRO5, which illustrates the indexes' great stability. The R-index increased in all cases because the h-core now includes more citations. Thus, the sum of citations within the core increased and so did R.

In Table 7, we present one example of the aforementioned citation concentration (80-5-5-5-5 scenario) for PRO5, PRO09, PRO10 and PRO11.

In Table 8, we present the indexes' values for all five scenarios of citation concentration. We do not present the p-index value because it remains the same when we redistribute citations, as previously explained.

Table 6: Dispersion of citations.

-	PRO05	PRO09	PRO10	PRO11
	2019	2019	2019	2019
1	43	15	24	10
2 to 6	24	6	12	6
7 to 11	24	1	12	1
12	24	1	12	-
13	24	1	1	-
14	24	1	1	-
15 to 24	24	-	-	-
25 to 46	1	-	-	-
Publications	46	14	14	11
Citations	617	53	158	45
h	24	6	12	6
g	24	6	12	6
A	24,8	7,5	13,0	6,7
R	24,4	6,7	12,5	6,3
ψ	34,0	9,0	17,0	8,0
p	20,2	5,9	12,1	5,7
z	16,6	4,6	11,4	4,9

Table 7: Concentration of citations –80-5-5-5-5 scenario.

publication order	PRO05	PRO09	PRO10	PRO11
	2019	2019	2019	2019
1	460	36	121	31
2	29	2	7	2
3	29	2	7	2
4	29	2	7	2
5	29	2	7	2
6 to 11	1	1	1	1
12 to 14	1	1	1	-
15 to 46	1	-	-	-

Table 8: Values of the h, g, A, R, ψ, p and z indexes for all scenarios.

	Index	PRO05	PRO09	PRO10	PRO11
80-5-5-5-5	h	5	2	5	2
	g	24	6	12	6
	A	115,20	19,00	29,80	16,50
	R	24,00	6,16	12,21	5,74
	ψ	34	9	17	8
	z	6,83	3,12	5,98	3,25
20-20-20-20-20	h	5	5	5	5
	g	24	6	12	6
	A	115,20	8,80	29,80	7,80
	R	24,00	6,63	12,21	6,24
	ψ	34	9	17	8
	z	10,10	4,66	8,94	4,78
30-30-30-5-5	h	5	3	5	3
	g	24	6	12	6
	A	115,20	13,33	29,80	12,00
	R	24,00	6,32	12,21	6,00
	ψ	34,00	9,00	17,00	8,00
	z	9,09	4,18	8,01	4,25
50- 30-10-5-5	h	5	3	5	3
	g	24	6	12	6
	A	115,20	13,33	29,80	12,00
	R	24,00	6,32	12,21	6,00
	ψ	34	9	17	8
	z	8,35	3,80	7,36	3,90
60-20-10-5-5	h	5	3	5	3
	g	24	6	12	6
	A	115,20	13,33	29,80	11,67
	R	24,00	6,32	12,21	5,92
	ψ	34	9	17	8
	z	7,92	3,63	6,98	3,79

From Table 8, we may observe that the *h*-index was greatly reduced for all professors, even when compared to the original data. This index varied the most across scenarios, indicating that it is very dependent on the citation distribution. The z-index also varied significantly, presenting the smallest values for the 80-5-5-5-5 scenario and the greatest values for the uniform scenario (20-20-20-20-20). However, even in the latter scenario, the z-index presented smaller values than in Table 6. This is another indication that z depends on the citation distribution and that it values dissipated distributions, though less than h.

The R-index increased, compared to the original data, because the sum of citations within the h-cores increased, even though the number of publications in the h-cores reduced. This index, as well as the A-index, presented practically the same value for all five scenarios.

The g and ψ-index remained exactly the same as in Table 6 (dissipated distribution) for all scenarios, which, again, illustrates their great stability, being little dependent on the citation distribution. In other words, the citations that a given researcher receives generally benefit his/her g and ψ-index, almost regardless of which publication those citations refer to.

CONCLUSION

The present article presented the *h*-index, proposed by Hirsch,[1] as well as h-type indexes that increase sensitivity to widely cited publications, namely the g, A, R and ψ indexes. For a more thorough analysis, we also compared them to the p-index, which measures a researcher's activity and excellence and to the z-index, which combines quantity, quality and consistency.

First, we analyzed the behavior of these indices, in light of increases in the researchers' production level. For that, we considered a case study based on the academic output of professors in the Production Engineering postgraduate course at UFF, in 2016 and in 2019. We found that the *h*-index varied the most over this time period, for professors who had h = 1 in 2016. For those who presented higher *h*-index values in 2016, the *h*-index relative increases were much smaller. On the other hand, the R, ψ, z and p indexes presented more similar relative increases, whereas g stood between both, varying less than the *h*-index, though more than the others. We did not deepen our analysis for the A index, because its results were inconsistent, compared to the other indexes.

Then, we studied the stability of the indexes, in light of variations in the citation distribution, without modifying the total number of citations or cited publications. We found that the g and the ψ indexes are significantly more stable than all the others, remaining constant even with relevant variations, even though they do not ignore the citation distribution entirely. On the other hand, the *h*-index varied the most, presenting the highest values for dissipated distributions and the lowest values for concentrated distributions. The z-index presented similar results, considering this perspective, though its variations were smaller. As the p-index depends on the total number of citations and publications, it does not vary when we redistribute citations.

In the present article, we found many differences in the behavior of the indexes studied herein. For this reason, educational institutions and funding agencies that need to measure researchers' production, for multiple purposes, such as promotions, scholarships, research payments, among others, should be aware of these characteristics. Future works may adopt a different perspective, by studying some of these organizations and recommending one (or more) bibliometric index (or indexes) for them, based on their purposes. Other researches may study theoretical implications of the properties verified herein, particularly regarding the indexes' stability in light of changes in the citation distribution.

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CONFLICT OF INTEREST

The authors declare no conflict of interest.

REFERENCES

1. Hirsch JE. An index to quantify an individual's scientific research output. Proceedings of the National Academy of Sciences. 2005;102(46):16569-72.
2. Franceschini F, Maisano DA. Analysis of the Hirsch index's operational properties. European Journal of Operational Research. 2010;203(2):494-504.
3. Egghe L. The Hirsch index and related impact measures. Annual Review of Information Science and Technology. 2010;44(1):65-114.
4. Bornmann L, Daniel HD. What do we know about the h index?. Journal of the American Society for Information Science and Technology. 2007;58(9):1381-5.
5. Braun T, Glänzel W, Schubert A. A Hirsch-type index for journals. The Scientist. 2006;69(1):169-73.
6. Burrell QL. Formulae for the *h*-index: A lack of robustness in Lotkaian informetrics?. Journal of the American Society for Information Science and Technology. 2013;64(7):1504-14.
7. Harzing AW, DerWal VR. Google Scholar as a new source for citation analysis. Ethics in Science and Environmental Politics. 2008;8(1):61-73.
8. Norris M, Oppenheim C. The *h*-index: A broad review of a new bibliometric indicator. Journal of Documentation. 2010;66:681-705.
9. Panaretos J, Malesios C. Assessing scientific research performance and impact with single indices. Scientometrics. 2009;81(3):635-70.
10. Alonso S, Cabrerizo FJ, Herrera-Viedma E, Herrera F. Hg-Index: A new index to characterize the scientific output of researchers based on the h- and g-indices. Scientometrics. 2010;82(2):391-400.
11. Anderson T, Hankin R, Killworth P. Beyond the Durfee square: Enhancing the h- index to score total publication output. Scientometrics. 2008;76(3):577-88.
12. Batista PD, Campiteli MG, Kinouchi O. Is it possible to compare researchers with different scientific interests?. Scientometrics. 2006;68(1):179-89.
13. Cabrerizo FJ, Alonso S, Herrera-Viedma E, Herrera F. q2-Index: Quantitative and qualitative evaluation based on the number and impact of papers in the Hirsch core. Journal of Informetrics. 2010;4(1):23-8.
14. Egghe L, Rousseau R. An *h*-index weighted by citation impact. Information Processing and Management. 2008;44(2):770-80.
15. Schreiber M. A modification of the *h*-index: The hm-index accounts for multiauthored manuscripts. Journal of Informetrics. 2008;2(3):211-6.
16. Tol RS. The h-index and its alternatives: An application to the 100 most prolific economists. Scientometrics. 2009;80(2):317-24.
17. Vinkler P. The π-index: A new indicator for assessing scientific impact. Journal of Information Science. 2009;35(5):602-12.
18. Vinkler P. The π_v-index: A new indicator to characterize the impact of journals. Scientometrics. 2010;82(3):461-75.
19. Zhang C. The *h'-index*, effectively improving the *h*-index based on the citations distribution. PLoS One. 2013;8(4):e59912.
20. Hirsch J. An index to quantify an individual's scientific research output that takes into account the effect of multiple coauthorship?. Scientometrics. 2010;85(3):741-54.
21. Hirsch J. E. h: An index to quantify an individual's scientific leadership?. Scientometrics. 2019;118(2):673-86.
22. Wildgaard L, Schneider JW, Larsen B. A review of the characteristics of 108 author-level bibliometric indicators?. Scientometrics. 2014;101(1):125-58.
23. Egghe L. An improvement of the *h*-index: The g-index. ISSI Newsletter. 2006;2(1):8-9.
24. Jin B. *H*-index: An evaluation indicator proposed by scientist. Science Focus. 2006;1(1):8-9.
25. Jin B, Liang L, Rousseau R, Egghe L. The R- and AR-indices: Complementing the *h*-index. Chinese Science Bulletin. 2007;52(6):855-63.
26. Lathabai HH. ψ-index: A new overall productivity index for actors of science and technology. Journal of Informetrics. 2020;14(4):101096.
27. Prathap G. Is there a place for a mock *h*-index?. Scientometrics. 2010a;84(1):153-65.
28. Prathap G. The 100 most prolific economists using the p-index. Scientometrics. 2010b;84(1):167-72.
29. Prathap G. The Zynergy-index and the Formula for the *h*-index. Journal of the Association for Information Science and Technology. 2014;65(2):426-7.
30. Bihui J. The AR-index: Complementing the *h*-index. ISSI Newsletter. 2007;3:6.
31. Schreiber M. Revisiting the g-index: The average number of citations in the g-core. Journal of American Society for Information Science and Technology. 2010;61(1):169-74.
32. Brandão LC, Soares DE, Mello JCCB. Comparative Analysis between Bibliometric Indicators. In: XLVII Simpósio Brasileiro De Pesquisa Operacional, 2015. Porto de Galinhas, PE. Anais[…] (Annals…). Rio de Janeiro: Sobrapo; 2015;142277.
33. Prathap G. The Energy–Exergy–Entropy (or EEE) sequences in bibliometric assessment. Scientometrics. 2011a;87(3):515-24.
34. Prathap G. Quasity, when quantity has a quality all of its own— toward a theory of performance. Scientometrics. 2011b;88(2):555-62.
35. DeCastro RJ, DosSantos CR, DeSoares MJCCB. Evaluation of the academic production of teachers of the graduate engineering courses of the Universidade Federal Fluminense using the DEA model and *h*-index. Target: Evaluation. 2017;9:408-32.
36. Brandão LC, DeSoares Mello, J.C.C.B. A multi-criteria approach to the *h*-index. European Journal of Operational Research. 2019;276(1):357-63.
37. Lando T, Bertolli-Basotti L. A new bibliometric index based on the shape of the citation distribution. PlosOne. 2014;9(12):e115962.